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CHEMICAL REACTION EFFECTS ON MHD FLOW PAST A LINEARLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF THERMAL RADIATION

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An exact solution of first order chemical reaction effects on a radiative flow past a linearly accelerated infinite isothermal vertical plate with variable mass diffusion, under the action of a transversely applied magnetic field has been presented. The plate temperature is raised linearly with time and the concentration level near the plate is also raised to C'_w linearly with time. The dimensionless governing equations are tackled using the Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters such as the magnetic field parameter, radiation parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that velocity increases with decreasing magnetic field parameter or radiation parameter. But the trend is just reversed with respect to the chemical reaction parameter.

Key words: accelerated, isothermal, radiation, vertical plate, heat and mass transfer, magnetic field, chemical reaction.

1. Introduction

Thermal radiation is an important factor in the thermodynamic analysis of many high temperature systems like solar collectors, boilers and furnaces. The simultaneous effect of heat and mass transfer in the presence of thermal radiation plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, cooling of towers, gas turbines and various propulsion device for aircraft, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. England and Emery (1969) studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar (1996). The governing equations were solved analytically. Das *et al.* (1996) analyzed radiation effects on the flow past an impulsively started infinite isothermal vertical plate.

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in a solution.

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Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al.* (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction were studied by Das *et al.* (1999). The dimensionless governing equations were solved by the usual Laplace-transform technique.

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil,and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in motion of earth core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Gupta *et al.* (1971) studied free convection on the flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on the flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of a magnetic field were studied by Raptis *et al*. (1981). MHD effects on the flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate were studied by Raptis and Singh (1981). Mass transfer effects on the flow past an uniformly accelerated vertical plate were studied by Soundalgekar (1982). Again, mass transfer effects on the flow past an accelerated vertical plate with uniform heat flux were analyzed by Singh and Singh (1983). Basanth and Prasad (1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Recently, Muthucumaraswamy *et al.* (2011) studied an exact solution of a hydromagnetic flow past an accelerated isothermal vertical plate in the presence of variable mass diffusion. Hence, it is proposed to study thermal radiation and MHD effects on an unsteady flow past a linearly accelerated infinite vertical plate with variable temperature and mass difusion in the presence of a chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error functions.

2. Mathematical formulation

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ is studied. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$, in its own plane against the gravitational field and the temperature from the plate is raised linearly with time and the concentration level near the plate is also raised to C'_w. The plate is also subjected to a uniform transverse magnetic field of strength B_0. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \tag{2.1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l\left(C' - C'_\infty\right). \tag{2.3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

The prescribed initial and boundary conditions are as follows

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all} & y, t' \le 0, \\ t' > 0: & u = \dfrac{u_0^3}{\nu} t', & T = T_\infty + \left(T_w - T_\infty\right) A t', & C' = C'_w & \text{at} & y = 0, \\ & u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as} & y \to \infty \end{array} \tag{2.4}$$

where, $A = \dfrac{u_0^2}{v}$.

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \qquad t = \frac{t' u_0^2}{\nu}, \qquad Y = \frac{y u_0}{\nu}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\mathrm{Gr} = \frac{g\beta\nu\left(T_w - T_\infty\right)}{u_0^3}, \qquad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \qquad \mathrm{Gc} = \frac{\nu g \beta^*\left(C'_w - C'_\infty\right)}{u_0^3}, \tag{2.5}$$

$$R = \frac{16 a^* v^2 \sigma T_\infty^3}{k u_0^2}, \qquad \Pr = \frac{\mu C_p}{k}, \qquad \mathrm{Sc} = \frac{v}{D}, \qquad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \qquad K = \frac{v K_l}{u_0^2}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \, \sigma\left(T_\infty^4 - T^4\right). \tag{2.6}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4. \tag{2.7}$$

By using Eqs (2.6) and (2.7), Eq.(2.2) reduces to

$$\rho\, C_p\,\frac{\partial T}{\partial t'}=k\,\frac{\partial^2 T}{\partial y^2}+16a^*\sigma T_\infty^3\left(T_\infty-T\right), \tag{2.8}$$

in Eqs (2.1) to (2.4), leads to

$$\frac{\partial U}{\partial t}=\mathrm{Gr}\,\theta+\mathrm{Gc}\,C+\frac{\partial^2 U}{\partial Y^2}-MU, \tag{2.9}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial Y^2}-\frac{R}{\mathrm{Pr}}\theta, \tag{2.10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2}-KC. \tag{2.11}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & U=0, & \theta=0, & C=0 & \text{for all} & Y,t\le 0,\\ t>0: & U=t, & \theta=t, & C=1 & \text{at} & Y=0,\\ & U\to 0, & \theta\to 0, & C\to 0 & \text{as} & Y\to\infty. \end{array} \tag{2.12}$$

3. Method of solution

The resulting solutions are in terms of exponential and complementary error function. The relation between the error function and its complementary error function is as follows

erfc(x) =1-erfc(x).

The dimensionless governing Eqs (2.9) to (2.11), subject to the initial and boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta=\frac{t}{2}\Big[\exp\left(2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Pr}}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{at}\right)\Big]+ \tag{3.1}$$

$$-\frac{\eta\,\mathrm{Pr}\sqrt{t}}{2\sqrt{R}}\Big[\exp\left(-2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{at}\right)-\exp\left(2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Pr}}+\sqrt{at}\right)\Big],$$

$$C=\frac{1}{2}\Big[\exp\left(2\eta\sqrt{Kt\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{Kt}\right)+\exp\left(-2\eta\sqrt{Kt\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{Kt}\right)\Big], \tag{3.2}$$

$$
\begin{aligned}
U = & \left(\frac{t}{2}(1+2bd)+e\right)\begin{bmatrix}\exp\left(2\eta\sqrt{Mt}\right)\operatorname{erfc}\left(\eta+\sqrt{Mt}\right)+\\ +\exp\left(-2\eta\sqrt{Mt}\right)\operatorname{erfc}\left(\eta-\sqrt{Mt}\right)\end{bmatrix}+\\
& -\frac{bd\eta\sqrt{t}}{\sqrt{M}}\left[\exp\left(-2\eta\sqrt{Mt}\right)\operatorname{erfc}\left(\eta-\sqrt{Mt}\right)-\exp\left(2\eta\sqrt{Mt}\right)\operatorname{erfc}\left(\eta+\sqrt{Mt}\right)\right]+\\
& -e\exp(ct)\begin{bmatrix}\exp\left(2\eta\sqrt{(M+c)t}\right)\operatorname{erfc}\left(\eta+\sqrt{(M+c)t}\right)+\\ +\exp\left(-2\eta\sqrt{(M+c)t}\right)\operatorname{erfc}\left(\eta-\sqrt{(M+c)t}\right)\end{bmatrix}+\\
& -d\exp(bt)\begin{bmatrix}\exp\left(2\eta\sqrt{(M+b)t}\right)\operatorname{erfc}\left(\eta+\sqrt{(M+b)t}\right)+\\ +\exp\left(-2\eta\sqrt{(M+b)t}\right)\operatorname{erfc}\left(\eta-\sqrt{(M+b)t}\right)\end{bmatrix}+\\
& +d\exp(bt)\begin{bmatrix}\exp\left(2\eta\sqrt{\Pr(a+b)t}\right)\operatorname{erfc}\left(\eta\sqrt{\Pr}+\sqrt{(a+b)t}\right)+\\ +\exp\left(-2\eta\sqrt{\Pr(a+b)t}\right)\operatorname{erfc}\left(\eta\sqrt{\Pr}-\sqrt{(a+b)t}\right)\end{bmatrix}+\\
& +e\exp(ct)\begin{bmatrix}\exp\left(2\eta\sqrt{\mathrm{Sc}(K+c)t}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{(K+c)t}\right)+\\ +\exp\left(-2\eta\sqrt{\mathrm{Sc}(K+c)t}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{(K+c)t}\right)\end{bmatrix}+\\
& -e\left[\exp\left(2\eta\sqrt{Kt\mathrm{Sc}}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{Kt}\right)+\exp\left(-2\eta\sqrt{Kt\mathrm{Sc}}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{Kt}\right)\right]+\\
& -d(1+bt)\left[\exp\left(2\eta\sqrt{Rt}\right).\operatorname{erfc}\left(\eta\sqrt{\Pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right).\operatorname{erfc}\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\right]+\\
& +\frac{bd\eta\Pr\sqrt{t}}{\sqrt{R}}\left[\exp\left(-2\eta\sqrt{Rt}\right)\cdot\operatorname{erfc}\left(\eta\sqrt{\Pr}-\sqrt{at}\right)-\exp\left(2\eta\sqrt{Rt}\right)\cdot\operatorname{erfc}\left(\eta\sqrt{\Pr}+\sqrt{at}\right)\right]
\end{aligned}
\tag{3.3}
$$

where

$$
a=\frac{R}{\Pr},\qquad b=\frac{M-R}{\Pr-1},\qquad c=\frac{M-K\mathrm{Sc}}{\mathrm{Sc}-1},\qquad d=\frac{\mathrm{Gr}}{2b^2(1-\Pr)},\qquad e=\frac{\mathrm{Gc}}{2c(1-\mathrm{Sc})},
$$

and $\eta = Y/2\sqrt{t}$.

4. Discussion of results

To interpret the results for a better understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, K, R, M and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be *0.6* which corresponds to water-vapor. The value of the Prandtl number Pr is chosen such that it represents air (Pr = *0.71*).The numerical values of the velocity, temperature and concentration are computed for the above mentioned parameters.

The concentration profiles for different values of the chemical reaction parmeter (K=*0.2*, *2,5*), Sc=*0.6* and t=*0.2* are shown in Fig.1. The effect of the chemical reaction parameter plays an important role in the concentration field. It is observed that the plate concentration decreases with increasing values of the chemical reaction parameter. Figure 2 illustrates the effect of the concentration profiles for different values of the Schmidt number (Sc=*0.16*, *0.6*, *2.01*), K=*2* and t=*0.2*. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with the decreasing Schmidt number.

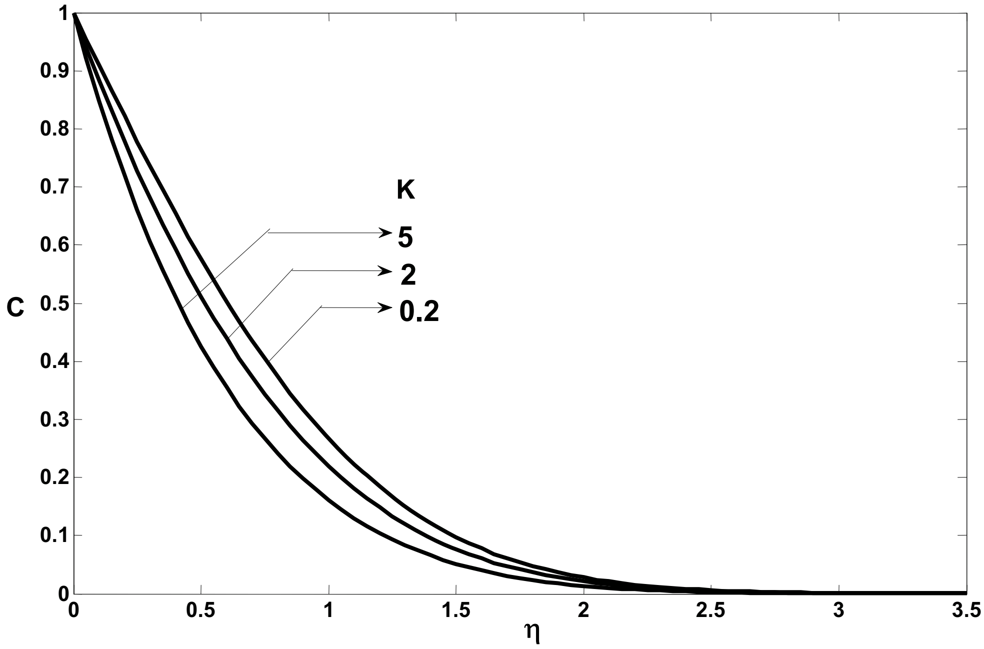

Fig.1. Concentration profiles for different values of *K*.

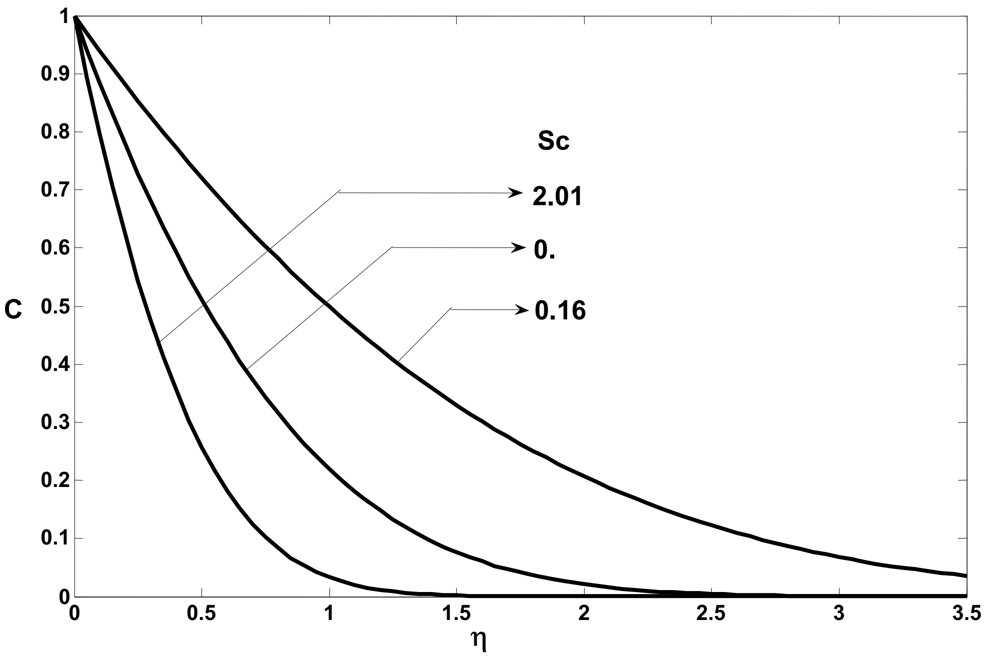

Fig.2. Concentration profiles for different values of Sc.

The temperature profiles are calculated for different values of the thermal radiation parameter (*R=0.2*, *2*, *5*) at time $t = 0.2$ and these are shown in Fig.3. It is observed that the temperature increases with the decreasing radiation parameter. The trend shows that there is a fall in plate temperature due to higher thermal radiation.

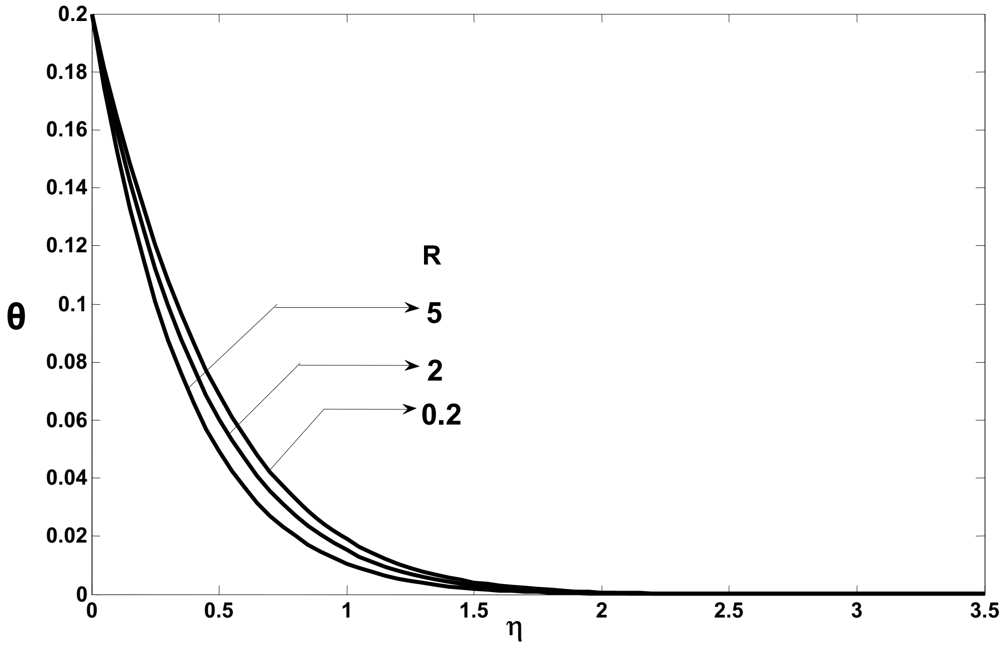

Fig.3. Temperature profile for different values of *R*.

The velocity profiles for different values of time $(t = 0.2, 0.4, 0.6)$, $K = 2, \mathrm{Gr} = 2, \mathrm{Gc} = 5, R = 5$ and $M = 2$ are studied and presented in Fig.4. It is observed that velocity increases with increasing values of time t.

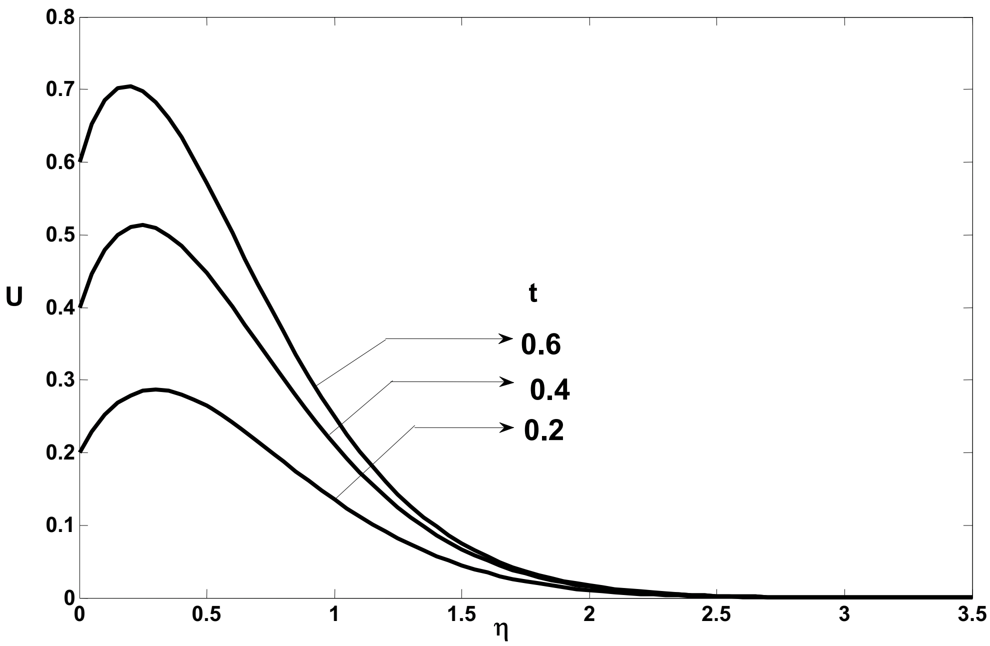

Fig.4. Velocity profiles for different values of *t*.

Figure 5 demonstrates the effect of velocity profiles for different values of the chemical reaction parameter ($K = 2, 5, 10$), $\mathrm{Gr} = 2, \mathrm{Gc} = 5, R = 5$, $M = 2$ and $t = 0.2$. It is observed that velocity increases with decreasing values of the chemical reaction parameter. The trend shows that there is a fall in velocity due to increasing values of the chemical reaction parameter.

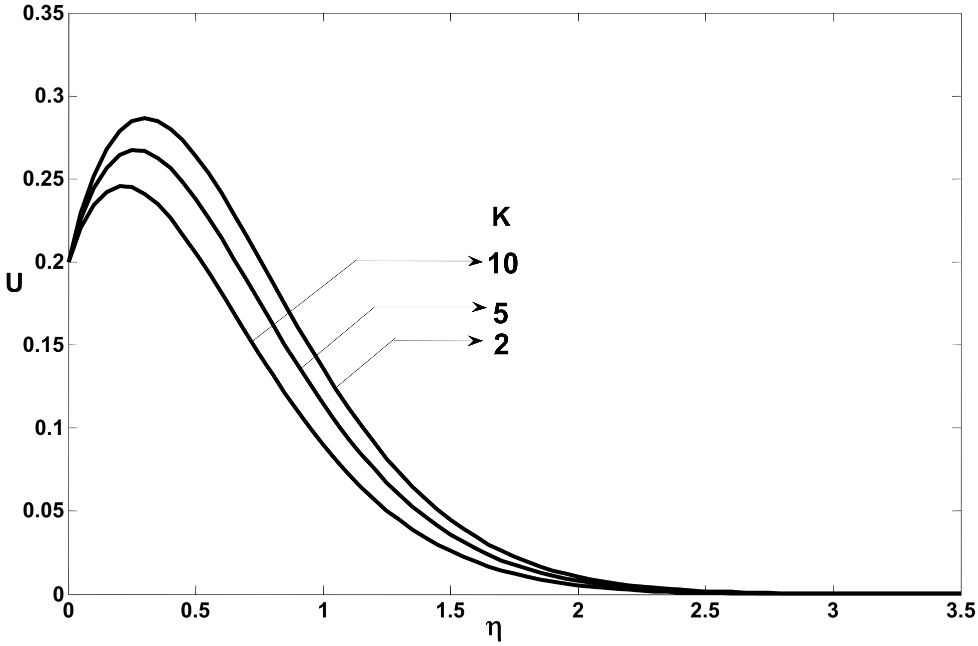

Fig.5. Velocity profiles for different values of *K*.

Figure 6 illustrates the effects of the magnetic field parameter on velocity when ($M = 2, 4, 7$), R=K=10, Gr=2, Gc=5 and $t = 0.2$. It is observed that velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow. Figure 7 demonstrates the effects of the radiation parameter on velocity when ($R = 2, 15, 45$), M=1, K=10, Gr=Gc=10 and $t = 0.2$. It is observed that velocity increases with the decreasing thermal radiation parameter. The trend shows that velocity is supressed due to higher thermal radiation.

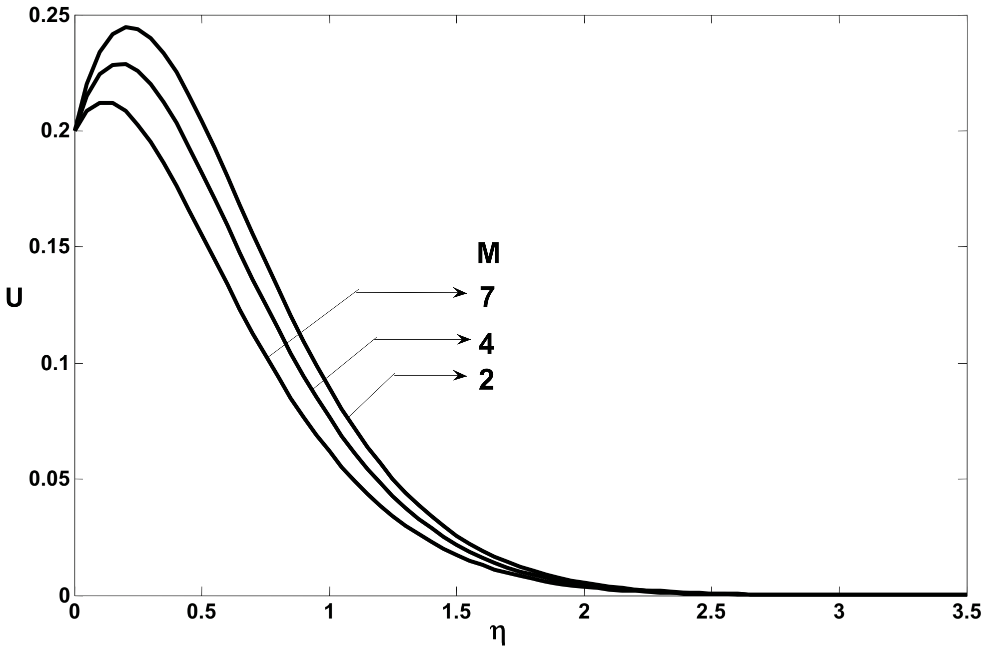

Fig.6. Velocity profiles for different values of *M*.

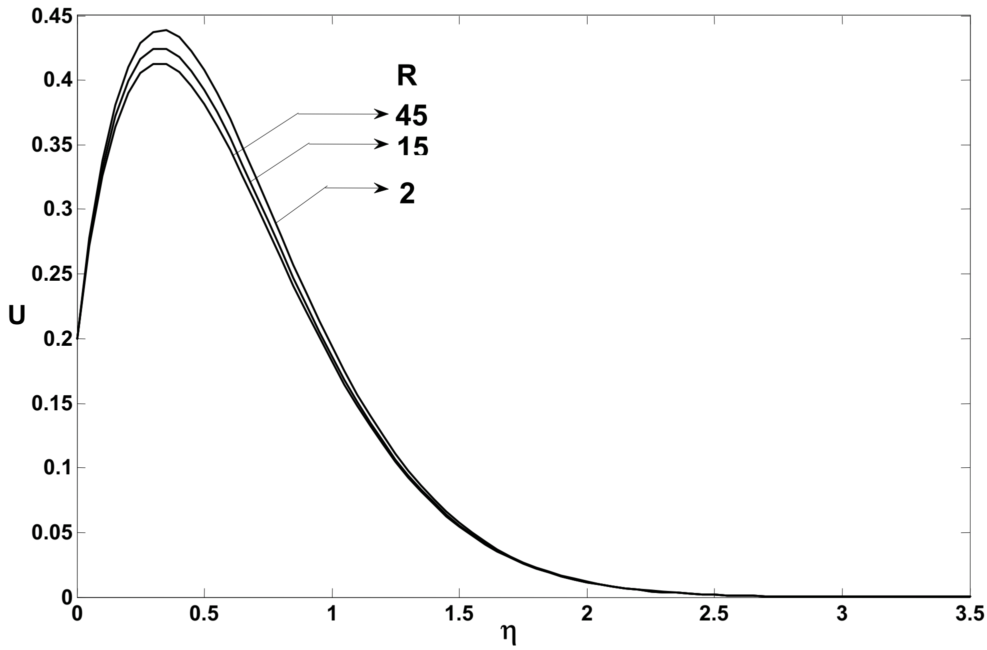

Fig.7. Velocity profiles for different values of *R*.

5. Concluding remarks

An exact solution of a thermal radiation and hydromagnetic flow past a linearly accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of a chemical reaction of first order is given. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters such as thermal Grashof number, mass Grashof number, chemical reaction parameter, radiation parameter, magnetic field parameter and t are studied graphically. The conclusions of the study are as follows:

(I) The temperature of the plate decreases with increasing values of the thermal radiation parameter.

(II) The concentration near the plate increases with decreasing values of the chemical reaction parameter or Schmidt number.

(III) Velocity increases with decreasing values of the magnetic field parameter or chemical reaction parameter or thermal radiation parameter. But the trend is just reversed with respect to time *t*.

Nomenclature

A – constant
a^* – absorption coefficient
C – dimensionless concentration
C' – species concentration in the fluid
C_p – specific heat at constant pressure
C'_w – concentration of the plate
C'_∞ – concentration of the fluid far away form the plate
D – mass diffusion coefficient
erfc – complementary error function
Gr – mass Grashof number
Gc – thermal Grashof number

g – accelerrated due to gravity
K – chemical reaction parameter
k – thermal conductivity
M – magnetic field parameter
Pr – Prandtl number
q_r – radiative heat flux in the y-direction
Sc – Schmidt number
T – temperature of the fluid near the plate
T_w – concentration of the plate
T_∞ – concentration of the fluid far away form the plate
t – dimensionless time
t' – time
U – dimensionless velocity
u – velocity of the fluid in the x-direction
u_0 – velocity of the plate
x – spatial coordinate along the plate
y – dimensionless coordinate axis normal to the plate
y' – coordinate axis normal to the plate m
β – volumetric coefficient of thermal expansion
β^* – volumetric coefficient of expansion with concentration
η – similarity parameter
θ – dimensionless temperature
μ – cofficient of viscosity
ρ – density of the fluid
τ – dimensionless skin-friction
υ – kinematic viscosity

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